MAC 2313 Calc III Practice Test #1 Ch 12–13

1.) $P(5,2,-3)$ $Q(1,-7,4)$

$$\boxed{\overrightarrow{PQ} = \langle -4,-9,7\rangle}$$

2.) center $(-4,2,-1)$ radius $=5$

$$\boxed{(x+4)^2+(y-2)^2+(z+1)^2=25}$$

3.) direction angles, α, β, γ $v=\langle -6,2,-3\rangle$

$|v| = \sqrt{(-6)^2+2^2+(-3)^2} = \sqrt{36+4+9} = \sqrt{49} = 7$

$\cos\alpha = -6/7$ $\cos\beta = 2/7$ $\cos\gamma = -\frac{3}{7}$

$\alpha = \cos^{-1}(-6/7)$ $\beta = \cos^{-1}(2/7)$ $\gamma = \cos^{-1}(-3/7)$

$\alpha = 2.600$ $\beta = 1.281$ $\gamma = 2.014$ (radians)
$\alpha = 148.997°$ $\beta = 73.398°$ $\gamma = 115.377°$ (degrees)

$\hat{a} = \langle -2,4,6\rangle$ $\hat{b} = \langle -1,2,-3\rangle$ $\hat{c} = \langle 1,3,-5\rangle$

4.) $3\hat{a}+2\hat{b}-\hat{c} = \langle -6,12,18\rangle + \langle -2,4,-6\rangle + \langle -1,-3,5\rangle$

$= \boxed{\langle -9,13,17\rangle}$

5.) $|\hat{b}| = \sqrt{(-1)^2+2^2+(-3)^2} = \sqrt{1+4+9} = \boxed{\sqrt{14}}$

6.) $|\hat{c}| = \sqrt{1^2+3^2+(-5)^2} = \sqrt{1+9+25} = \sqrt{35}$ unit vector $(u) = \frac{1}{|c|}\cdot c$

$u = \frac{1}{\sqrt{35}}\langle 1,3,-5\rangle = \boxed{\left\langle \frac{1}{\sqrt{35}}, \frac{3}{\sqrt{35}}, \frac{-5}{\sqrt{35}}\right\rangle}$

7.) $\hat{a}\cdot\hat{b} = -2\cdot -1 + 4\cdot 2 + 6\cdot -3$
$= 2+8-18$

$$\boxed{\hat{a}\cdot\hat{b} = -8}$$

8.) $\hat{a} \times \hat{c}$ $\quad \hat{a} = \langle -2, 4, 6 \rangle \quad \hat{c} = \langle 1, 3, -5 \rangle$

$$\hat{a} \times \hat{c} = \begin{vmatrix} i & j & k \\ -2 & 4 & 6 \\ 1 & 3 & -5 \end{vmatrix} = i(-20-18) - j(10-6) + k(-6-4)$$

$$= -38i - 4j - 10k = \boxed{\langle -38, -4, -10 \rangle}$$

9.) Area of parallelogram formed by $\hat{a}$ and $\hat{c}$

$$|\hat{a} \times \hat{c}| = |\langle -38, -4, -10 \rangle| = \sqrt{(-38)^2 + (-4)^2 + (-10)^2} = \sqrt{1560} = \boxed{2\sqrt{390}}$$

10.) The angle between $\hat{b}$ and $\hat{c}$. $\quad b = \langle -1, 2, -3 \rangle \quad \hat{c} = \langle 1, 3, -5 \rangle$

$\hat{b} \cdot \hat{c} = -1 \cdot 1 + 2 \cdot 3 + -3 \cdot -5 = -1 + 6 + 15 = 20$

$|\hat{b}| = \sqrt{(-1)^2 + 2^2 + (-3)^2} = \sqrt{14} \qquad |\hat{c}| = \sqrt{1^2 + 3^2 + (-5)^2} = \sqrt{35}$

$$\cos(\theta) = \frac{\hat{b} \cdot \hat{c}}{|\hat{b}||\hat{c}|} \qquad \cos\theta = \frac{20}{\sqrt{14} \cdot \sqrt{35}} = \frac{20}{\sqrt{490}} \qquad \theta = \cos^{-1}\left(\frac{20}{\sqrt{490}}\right)$$

$\boxed{\theta = 25.377^\circ \text{ or } \; \theta = .443 \text{ radians}}$

11.) $$\text{comp}_a b = \frac{\hat{a} \cdot \hat{b}}{|\hat{a}|} = \frac{2 + 8 - 18}{\sqrt{(-2)^2 + 4^2 + 6^2}} = \boxed{\frac{-8}{\sqrt{56}}}$$

12.) $$\text{proj}_a b = \frac{\hat{a} \cdot \hat{b}}{|\hat{a}|} \cdot \frac{\hat{a}}{|\hat{a}|} = \frac{-8}{\sqrt{56}} \cdot \frac{\langle -2, 4, 6 \rangle}{\sqrt{56}} = \frac{-8}{56} \langle -2, 4, 6 \rangle = \boxed{\left\langle \frac{2}{7}, \frac{-4}{7}, \frac{-6}{7} \right\rangle}$$

13.) Vector equation $(2, -4, 1)$ and $(7, 1, -3)$

$\hat{r} = \hat{r}_0 + t\hat{v}$ $\quad \hat{v} = \text{direction vector} = \langle 7-2, 1+4, -3-1 \rangle = \langle 5, 5, -4 \rangle$

$$\boxed{\hat{r} = \langle 2, -4, 1 \rangle + t\langle 5, 5, -4 \rangle}$$

14.) plane equation $(1, -2, 3)$ parallel to $2x - 3y + 5z = 8$

$2x - 3y + 5z = D$ $\qquad \boxed{2x - 3y + 5z = 23}$

$2(1) - 3(-2) + 5(3) = D$

$2 + 6 + 15 = D$

$23 = D$

15.) $\vec{r}(t) = \langle t^3 - 2t, t^2 + 5, 3t \rangle$ $t = 1$ point $(-1, 6, 3)$

$\vec{r}\,'(t) = \langle 3t^2 - 2, 2t, 3 \rangle$ $\langle -1, 6, 3 \rangle + t\langle 1, 2, 3 \rangle$

$\vec{r}\,'(1) = \langle 1, 2, 3 \rangle$

$$\boxed{\begin{aligned} x &= -1 + t \\ y &= 6 + 2t \\ z &= 3 + 3t \end{aligned}}$$

16.) $y = x^4$ @ $(2, 16)$

$y' = 4x^3$ $K = \dfrac{|y''|}{[1 + (y')^2]^{3/2}} = \dfrac{12x^2}{[1 + (4x^3)^2]^{3/2}} = \dfrac{12x^2}{[1 + 16x^6]^{3/2}}$

$y'' = 12x^2$

$K(2) = \dfrac{12(2)^2}{[1 + 16(2)^6]^{3/2}} = \boxed{\dfrac{48}{[1025]^{3/2}} \approx .0015}$

17.) $\int_0^1 \langle 2t^2, 5t + 3, \sqrt{t} \rangle\, dt = \int_0^1 2t^2 dt, \int_0^1 5t + 3\, dt, \int_0^1 t^{1/2} dt$

$= \frac{2}{3}t^3\Big]_0^1 + \frac{5}{2}t^2 + 3t\Big]_0^1 + \frac{2}{3}t^{3/2}\Big]_0^1$

$$\boxed{\left\langle \frac{2}{3}, \frac{11}{2}, \frac{2}{3} \right\rangle}$$

18.) $r(t) = \langle 5\sin(t), -12\sin(t), 13\cos(t) \rangle$ $t = 0$ to $t = 7$

$r'(t) = \langle 5\cos(t), -12\cos(t), -13\sin(t) \rangle$

$|r'(t)| = \sqrt{(5\cos(t))^2 + (-12\cos(t))^2 + (-13\sin(t))^2} = \sqrt{25\cos^2(t) + 144\cos^2(t) + 169\sin^2(t)}$

$= \sqrt{169\cos^2(t) + 169\sin^2(t)} = \sqrt{169(\cos^2 t + \sin^2 t)} = 13$

$\int_0^7 13\,dt = 13t\Big]_0^7 = \boxed{91}$

19.) $r(t) = \langle 2t^2, -5t \rangle$ $v(t) = r'(t) = \boxed{\langle 4t, -5 \rangle}$

20.) $a(t) = v'(t) = \boxed{\langle 4, 0 \rangle}$

21.) speed: $|v'(t)| = \sqrt{(4t)^2+(-5)^2} = \boxed{\sqrt{16t^2+25}}$

22.) unit tangent vector $T(t) = \dfrac{r'(t)}{|r'(t)|} = \boxed{\dfrac{1}{\sqrt{16t^2+25}}\langle 4t, -5\rangle}$ or $\dfrac{\langle 4t, -5\rangle}{\sqrt{16t^2+25}}$

23.) $T(t) = (16t^2+25)^{-1/2}\langle 4t, -5\rangle$

$T'(t) = (16t^2+25)^{-1/2}\langle 4, 0\rangle + \langle 4t, -5\rangle\left(-\frac{1}{2}(16t^2+25)^{-3/2}(32t)\right)$ $\quad -16t$

$T'(t) = (16t^2+25)^{-1/2}\langle 4, 0\rangle + \langle -64t^2, 80t\rangle(16t^2+25)^{-3/2}$

$T'(t) = (16t^2+25)^{-3/2}\left[(16t^2+25)\langle 4, 0\rangle + \langle -64t^2, 80t\rangle\right]$

$= (16t^2+25)^{-3/2}\left[\langle 64t^2+100, 0\rangle + \langle -64t^2, 80t\rangle\right]$

$= \boxed{\dfrac{\langle 100, 80t\rangle}{(16t^2+25)^{3/2}}}$ or $\boxed{\dfrac{20}{(16t^2+25)^{3/2}}\langle 5, 4t\rangle}$

24.) $|T'(t)| = \dfrac{20}{(16t^2+25)^{3/2}}\sqrt{5^2+(4t)^2} = \dfrac{20}{(16t^2+25)^{3/2}}\sqrt{25+16t^2} = \dfrac{20}{(16t^2+25)^{3/2}}(16t^2+25)^{1/2}$

$|T'(t)| = \boxed{\dfrac{20}{(16t^2+25)}}$

25.) $N(t) = \dfrac{T'(t)}{|T'(t)|} = \dfrac{\frac{20}{(16t^2+25)^{3/2}}\langle 5, 4t\rangle}{\frac{20}{(16t^2+25)}} = \dfrac{\cancel{20}\langle 5, 4t\rangle}{(16t^2+25)^{3/2}} \cdot \dfrac{(16t^2+25)}{\cancel{20}}$

$= \boxed{\dfrac{\langle 5, 4t\rangle}{(16t^2+25)^{1/2}}}$

26.) $K = \dfrac{|T'(t)|}{|r'(t)|} = \dfrac{\frac{20}{(16t^2+25)}}{(16t^2+25)^{1/2}} = \dfrac{20}{(16t^2+25)} \cdot \dfrac{1}{(16t^2+25)^{1/2}} = \boxed{\dfrac{20}{(16t^2+25)^{3/2}}}$